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A Two Stage Slacks-Based Measure Model with Negative Outputs and Super Efficiency: A Case of the Commercial Banks in Taiwan

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Although negative performance for organizations are not unusual, they are only included in the data envelopment analysis by a few recent works. Furthermore, their solutions concern only the one-stage evaluation. As a result, the intermediate production information cannot be utilized for instrumental purposes. The present paper develops a two-stage slacks-based measure (SBM) model and a Super two-stage SBM model to handle negative outputs, intermediate production information and ranking data. Managerial and performance data of 24 commercial banks in Taiwan were used to demonstrate the models which resulted in findings as follows. First, greater size banks performed worse than smaller ones in terms of operation efficiency, however, they were more appealing than small banks in marketability efficiency. Second, financial holding banks outperformed their rivals of conventional banking at overall efficiency (operation and

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marketability). These detailed findings may not be available with a one-stage SBM model. The assessment due on the featured models endeavored further projection analysis which could be configured in a matrix to better reveal managerial implications for the inefficient banks as an instrumental guideline to achieve superiority.

***Keywords**: Network DEA, Performance Measurement, Black Box, Matrix Management*

I. Introduction

Although Farrell (1957) pioneered the data envelopment analysis (DEA) of production frontier and ensured deterministic non-parametric efficiency frontier formulated with mathematical programming approach to assess technical efficiency, it was until Charnes, Cooper, and Rhodes (1978) who generalized Farrell's measure of single output efficiency into multiple outputs that the DEA is consolidated. Henceforward, DEA as well as its siblings -- the CCR model (Charnes, Cooper, and Rhodes, 1978) and BCC model (Banker, Charnes, and Cooper, 1984) -- were widely applied in efficiency analyses for the public and private organizations (Lewin and Lovell, 1995).

The DEA model can be divided into two categories, that is, black box models and network models. Black box models indicate that the production system have regarded as a black box and have only discussed single stage evaluation process for evaluating decision making units (DMUs). Therefore, the CCR model and BCC model are belong to this category of DEA models. In contrast to the black box models, the network models have further divided the production system into multi-stage processes, each DMU is measured the efficiencies of the various stages and the overall efficiency. The two-stage DEA model may become the most used model among the network models because it can investigate the inside structure of a DMU, provide right insights to improve the misallocation of resources within the inefficiency DMU.

In addition to consideration of the impact of the immediate production between stages, the conventional DEA model must assume the input and output value to be positive when measuring the efficiency performance in DEA model. Therefore, the conventional DEA model cannot deal with negative values (Charnes *et al.*, 2013), many studies ignore banks with negative values (Luo, 2003). However, the financial institutions may incur losses on investing derivative financial products in a global economic downturn and therefore there are likely to be negative outputs in the production process (Tsang *et al.*, 2014). For example, the subprime mortgage crisis caused global financial crisis in August 2007, the Taiwan Financial Supervisory Commission show that the Taiwan financial market suffered huge losses in this period of time. In sum, many banks suffered negative profits and earnings per share (EPS) during the 2008-2009 global financial crises.

In fact, there is less two-stage DEA model for efficiency assessment that can handle negative data. Therefore, the first objective of this paper is to present a two-stage slacks-based measure (SBM) model that meets the two demands. This model is an extension of the two-stage model (Luo, 2003; Seiford and Zhu, 1999; Sexton and Lewis, 2003) and the SBM model (Tone, 2001). Secondly, DEA like assessments are often adopted to deal ranking data of efficient DMUs. To this type of data, we develop a super two-stage SBM model based on our two-stage SBM model that handles negative data, multi-stages and ranking data. The remainder of this paper is organized as follows: a literature review is in the following section. Section 3 introduces methodological models of the paper, containing SBM model and two-stage SBM model with negative data. Section 4 presents the empirical application with samples of banks. Conclusions are drawn in Section 5.

II. Literature Review

An important progress was lately made which relaxed a constraint of the traditional DEA where it allowed only positive values of inputs and outputs with respect to efficiency assessment. A simple method to handle the negative values is to transform the negative data into positive data (Lovell and Pastor, 1995; Seiford and Zhu, 2002). Although the transformation does not alter the efficient frontier, the inefficiency scores attached with the inefficient decision making units (DMUs) are already projected differently which are different from their original natures, referred as translation invariance (Charnes *et al.*, 1985). In other words, though the additive model is translation invariant, it is not unit invariant consequently the efficiency score is distorted by the units of measurement of the inputs and outputs.

While retaining the feature of translation invariance, the shortcomings of the additive model was addressed by weighted additive model (Lovell and Pastor, 1995) and the range adjusted measure (RAM) model (Cooper, Park, and Pastor, 1999). These models were superior in analysing the negative data in original forms. The negative data challenge was readdressed by Portela, Thanassoulis, and Simpson (2004) where they applied the directional distance function (Chambers, Chung, and Färe, 1996) to develop a range directional model (RDM) and was further turned into a generalized Farrell proportional distance function (Kerstens and Van de Woestyne, 2010). The merit of using the directional distance function is that it can take into account both input contractions and output improvements simultaneously to assess efficiency, which is similar to the non-oriented SBM model (Tone, 2001). These SBM alike models however were limited by the lack of dealing with non-positive (zero and negative) data. The SBM model was revised to form a modified slack-based measure (MSBM) by Sharp, Meng, and Liu (2007). This revision bridged another stream of DEA modeling with respect to the negative data problem as follows.

Another stream dealing with the negative data problem was proposed by Emrouznejad and colleagues (Emrouznejad *et al.*, 2010; Emrouznejad, Anouze, and Thanassoulis, 2010), a semi-oriented radial measure (SORM). By dismantling a variable into two, the positive values and the negative values, this SORM approach could treat the negative part of a variable in absolute value term as in positive format without arbitrary changes of origin. Nevertheless, the dismantling increased the number of variables and also subjected to the similar concerns of BCC model where the input and output information could not be taking into account simultaneously. Besides, for handling the negative data prior studies assumed the convexity constraint, i.e., variable returns to scale (VRS) (Ali and Seiford, 1990). In addition, the scope of conventional DEA is limited to performance improvement in the same context. Consequently, Chiu *et al.* (2014) proposed a context-dependent RAM model to provide a reliable means of exploring operational performance in different evaluative contexts and taking into consideration negative outputs. The model can effectively measure and rank the operational performance of 23 commercial banks in Taiwan in 2008. Tavana *et al.* (2017) indicated that many banking operations can be modelled as two-stage processes and then they propose a new dynamic range directional measure for two-stage DEA models that allows for negative data to evaluate the efficiency of 29 commercial bank branches of a US bank over a three-year period. The results showed that only five out of the 29 bank branches analyzed were overall efficient during the examined years.

In addition to the negative data challenge, the inclusion of information between stages is another emerging call from the practice. The aforementioned literatures however considered only on one-stage situation. They cannot diagnose the sources of inefficiency if they were from the processes. Traditional DEA assumes that production system is a black box that considers only one stage process. On the contrary, many production processes, such as the financial service sector, need more than one stage to reflect sufficient management information. Seiford and Zhu (1999) adopted the two-stage DEA method to assess the management performances of 55 commercial banks in the US, in which they divided the production process of commercial banks into two stages: profitability and marketability. The result showed that larger banks had better management performance in profitability, whereas smaller banks had better performance in marketability. Zhu (2000) proceeded to analyze the financial performance of the best 500 companies, ranked by *Fortune* magazine, with a three-stage DEA. The results showed that the top-ranked companies in revenue did not necessarily have the top-ranked performance in terms of profitability and marketability, and most companies displayed not only serious technical inefficiencies, but also serious scale inefficiencies. Luo (2003) pointed out that the market value of shares also reflects the real value of the corporation, and thus the evaluation of market efficiency is indispensable. So, the items of assessment are critical.

An innovative two-stage network technology approach is applied to analyse the cost efficiency of Turkish banks and the empirical result found that the two-stage network system is superior to traditional DEA in the banking sector and confirmed that the traditional DEA approach overestimates efficiency scores (Fukuyama and Matousek, 2011). Chiu *et al.* (2016) develop a new approach to explore the sources of metafrontier inefficiency for managers of various banks in a two-stage network system. They demonstrated that foreign banks do not have high efficiency in developed countries, the production and operational inefficiencies of the metafrontier are derived from the managerial inefficiency of domestic and foreign banks for the 2004-2009.

III. Methodology

The SBM model (Tone, 2001), which stemmed from the additive model (Charnes *et al.*, 1985), was proposed to utilize the slacks of inputs and outputs measures to assess the efficiency score of DMUs. This was a non-radial model which assessed the Pareto-Koopmans efficiency of DMUs when all slacks were zero. Because the SBM model took into account only the unit invariance, i.e., excluding the translation invariance, the model was limited for it could only handle positive data of inputs and outputs measures of efficiency. For negative data to be included, the inclusion of the translation invariance is a key step. Therefore, Portela, Thanassoulis, and Simpson (2004) proposed a revised RDM model which included the SP range to the directional distance function to handle negative data. The SP range was further adopted in the MSBM model (Sharp, Meng, and Liu, 2007) to modify the objective function of SBM model and resolved the exclusion of translation invariance in the SBM model as follows. The MSBM model evaluated the efficiency of the $\mathbf{DMU_k}$ by solving the following linear programs:

$$
\begin{aligned}
&\textbf{Min} \quad \rho_k = \frac{1-\sum_{i=1}^{m}\frac{w_i s_i^{-}}{P_{ik}^{-}}}{1+\sum_{r=1}^{s}\frac{v_r s_r^{+}}{P_{rk}^{+}}} \\
&s.t. \quad \mathrm{x}_{ik} = \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^{-}, \quad i=1,\ldots,m, \\
&\qquad \mathrm{y}_{rk} = \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^{+}, \quad r=1,\ldots,s, \\
&\qquad \sum_{j=1}^{n}\lambda_j = 1, \qquad j=1,\ldots,n, \\
&\qquad \sum_{i=1}^{m} w_i = 1, \quad \sum_{r=1}^{s} v_r = 1, \\
&\qquad \lambda_j, w_i, v_r, s_i^{-}, s_r^{+} \geq 0,
\end{aligned}
\tag{1}
$$

where w_i and v_r represent the slack weight of *i-th* inputs, *r-th* outputs, the SP range of $\mathbf{DMU_k}$ is defined by $P_{ik}^{-} = x_{ik} - \min_{j=1,\ldots,n} x_{ij}$, $P_{rk}^{+} = \max_{j=1,\ldots,n} y_{rj} - y_{rk}$. The above model is not only unit invariance but also translation invariance and can be applicable to the realistic case for handling negative data. However, this model was limited when the SP range was zero; when the zero happened the corresponding terms were dropped from the numerator and the denominator. Accordingly, we have the following model setting that there are n DMUs using m_1 positive inputs and m_2 negative inputs to produce s_1 positive outputs and s_2 negative outputs with $\min_{j=1,\dots,n} x^p_j > 0$, $\min_{j=1,\dots,n} x^n_j < 0$, $\min_{j=1,\dots,n} y^p_j > 0$ and $\min_{j=1,\dots,n} y^n_j < 0$.

An objective function of SBM model is derived as follow:

$$
\textbf{Min} \ \ \rho_k = \frac{1 - \dfrac{1}{m_1 + m_2}\left(\displaystyle\sum_{i=1}^{m_1} \frac{s_i^p}{x^p_{ik}} + \sum_{i=1}^{m_2} \frac{s_i^n}{R_i^n}\left(1 + \frac{x^n_{ik}}{\underline{x}^n_i}\right)\right)}{1 + \dfrac{1}{s_1 + s_2}\left(\displaystyle\sum_{r=1}^{s_1} \frac{s_r^p}{y^p_{rk}} + \sum_{r=1}^{s_2} \frac{s_r^n}{R_r^n}\left(1 - \frac{y^n_{rk}}{\overline{y}^n_r}\right)\right)}
$$

$$
\begin{aligned}
\textbf{s.t.} \quad & x^p_{ik} = \sum_{j=1}^{n} x^p_{ij}\lambda_j + s_i^p, && i = 1,\dots,m_1, \\
& x^n_{ik} = \sum_{j=1}^{n} x^n_{ij}\lambda_j + s_i^n, && i = 1,\dots,m_2, \\
& y^p_{rk} = \sum_{j=1}^{n} y^p_{rj}\lambda_j - s_r^p, && r = 1,\dots,s_1, \\
& y^n_{rk} = \sum_{j=1}^{n} y^n_{rj}\lambda_j - s_r^n, && r = 1,\dots,s_2, \\
& \sum_{j=1}^{n} \lambda_j = 1, && j = 1,\dots,n, \\
& \lambda_j, s_i^p, s_i^n, s_r^p, s_r^n \ge 0,
\end{aligned} \tag{2}
$$

where $R_i^n = \max_{j=1,\dots,n} x^n_{ij} - \min_{j=1,\dots,n} x^n_{ij}$, $R_r^n = \max_{j=1,\dots,n} y^n_{rj} - \min_{j=1,\dots,n} y^n_{rj}$, and $\underline{x}^n_i = \min_{j=1,\dots,n} x^n_{ij}$, $\overline{y}^n_r = \max_{j=1,\dots,n} y^n_{rj}$. Note that the function is assumed VRS form in order to satisfy translation invariance and guarantee the pure technical efficiency between zero and unit. The merit of this model is it can assess operating efficiency with negative data where the efficient score contains unit invariance and translation invariance.

3.1 Two-Stage SBM Model with Negative Data

While conventional DEA is capable of revealing the efficiency performance of DMUs via the efficiency information of inputs and outputs, it is deficient in concerning the information of DMUs' managerial activities in proceeding. In other words, conventional DEA cannot be used to detect managerial efficiency with respect to the stages. To solve this problem, Tone and Tsutsui (2009) proposed the network slacks-based measure (NSBM) model based on the SBM model to include the intermediate information. Nevertheless, the NSBM model subjected to the same limit of the SBM model where the translation invariance was excluded, as a result, the negative outputs were left out of modeling. Given that the two-stage SBM model is a special case of the NSBM model, by including the negative data in the two-stage SBM model, we should arrive a solution to handle both the negative datas and the intermediate information.

In view of this, this study revises the constraint of equation (2) to employs the two-stage SBM model. Given n DMUs (DMU_j, where $j \in R^n$) on m_1 positive inputs ($x_j \in R^{m_1}$), m_2 negative inputs ($x_j \in R^{m_2}$), l intermediate products ($z_j \in R^l$), s_1 positive outputs ($y_j^p \in R^{s_1}$), and s_2 negative outputs ($y_j^n \in R^{s_2}$) for the two stages, let's assume $\min_{j=1,\ldots,n} x_j^p > 0$, $\min_{j=1,\ldots,n} x_j^n < 0$, $\min_{j=1,\ldots,n} z_j > 0$, $\min_{j=1,\ldots,n} y_j^p > 0$ and $\min_{j=1,\ldots,n} y_j^n < 0$, then a two-stage SBM assessment on the k^th^ $\mathbf{DMU_k}$ is derived by solving the following linear programs:

$$\textbf{Min}\ \rho_k^* = \frac{w_1 \times \left[1 - \frac{1}{m_1+m_2}\left(\sum_{i=1}^{m_1}\frac{s_i^p}{x_{ik}^p} + \sum_{i=1}^{m_2}\frac{s_i^n}{R_i^n}\left(1+\frac{x_{ik}^n}{\underline{x}_i^n}\right)\right)\right] + w_2}{w_1 + w_2 \times \left[1 + \frac{1}{s_1+s_2}\left(\sum_{r=1}^{s_1}\frac{s_r^p}{y_{rk}^p} + \sum_{r=1}^{s_2}\frac{s_r^n}{R_r^n}\left(1-\frac{y_{rk}^n}{\overline{y}_r^n}\right)\right)\right]}$$

$$\begin{aligned}
\textbf{s.t.}\quad & x_{ik}^p = \sum_{j=1}^{n} x_{ij}^p \lambda_j + s_i^p, \quad i=1,\ldots,m_1,\\
& x_{ik}^n = \sum_{j=1}^{n} x_{ij}^n \lambda_j + s_i^n, \quad i=1,\ldots,m_2,\\
& \sum_{j=1}^{n}\left(\lambda_j - \mu_j\right) z_{bj} = 0, \quad b=1,\ldots,l,\\
& y_{rk}^p = \sum_{j=1}^{n} y_{rj}^p \mu_j - s_r^p, \quad r=1,\ldots,s_1,\\
& y_{rk}^n = \sum_{j=1}^{n} y_{rj}^n \mu_j - s_r^n, \quad r=1,\ldots,s_2,\\
& \sum_{j=1}^{n}\lambda_j = 1, \sum_{j=1}^{n}\mu_j = 1 \quad j=1,\ldots,n,\\
& w_1 + w_2 = 1,\\
& \lambda_j, \mu_j, w_1, w_2, s_i^p, s_i^n, s_r^p, s_r^n \geq 0,
\end{aligned} \tag{3}$$

where w_1 and w_2 represent the relative weight of the first and second stages, λ_j and μ_j are the intensity variables corresponding to the first and second stages; s_i^p, s_i^n, s_r^p and s_r^n are the slacks of positive input and negative input terms, positive output and negative output terms; R_i^n and R_r^n is the range of negative input and negative output terms, where $R_i^n = \max_{j=1,\ldots,n} x_{ij}^n - \min_{j=1,\ldots,n} x_{ij}^n$, $R_r^n = \max_{j=1,\ldots,n} y_{rj}^n - \min_{j=1,\ldots,n} y_{rj}^n$, and $\underline{x}_i^n = \min_{j=1,\ldots,n} x_{ij}^n$, $\overline{y}_r^n = \max_{j=1,\ldots,n} y_{rj}^n$. In addition, the intermediate products between the first and the second stage are assumed changing freely. Consequently, if the overall efficiency (ρ_k^*) is equal to unit, the $\mathbf{DMU_k}$ located on the performance frontier in both stages with respect to s_i^p, s_i^n, s_r^p and s_r^n are equal to zero; otherwise, the score (ρ_k^*) could be between zero and unit. The score (ρ_k^1) of the first stage is equal to $1-\frac{1}{m_1+m_2}\left(\sum_{i=1}^{m_1}\frac{s_i^p}{x_{ik}^p}+\sum_{i=1}^{m_2}\frac{s_i^n}{R_i^n}\left(1+\frac{x_{ik}^n}{\underline{x}_i^n}\right)\right)$, and the score (ρ_k^2) of the second stage is equal to $\frac{1}{\left[1+\frac{1}{s_1+s_2}\left(\sum_{r=1}^{s_1}\frac{s_r^p}{y_{rk}^p}+\sum_{r=1}^{s_2}\frac{s_r^n}{R_r^n}\left(1-\frac{y_{rk}^n}{\overline{y}_r^n}\right)\right)\right]}$.

3.2 Projection Analysis

An immediate interest is how the inefficient DMUs be instrumented to improve. To this end, the projection analysis below is proposed to reveal the opportunities for improvement in the light of the inputs, intermediate production, and outputs. Assume that the optimal solution of equation (3) is ($\lambda_j^*, s_{ik}^{p*}, s_{ik}^{n*}, s_{rk}^{p*}, s_{rk}^{n*}$), hence the projection positive inputs, negative inputs, intermediate products, positive outputs and negative outputs of **DMU$_\mathbf{k}$** can be expressed with equation (4):

$$\begin{aligned}
x_{ik}^{p*} &= x_{ik} - s_{ik}^{p*}, \quad i=1,\dots,m_1,\\
x_{ik}^{n*} &= x_{ik} - s_{ik}^{n*}, \quad i=1,\dots,m_2,\\
z_{bk}^{*} &= \lambda_j^* z_{bj}, \qquad b=1,\dots,l,\\
y_{rk}^{p*} &= y_{rk}^{p} + s_{rk}^{p*}, \quad r=1,\dots,s_1,\\
y_{rk}^{n*} &= y_{rk}^{n} + s_{rk}^{n*}, \quad r=1,\dots,s_2,
\end{aligned} \tag{4}$$

3.3 Super Two-Stage SBM Model with Negative Output

Furthermore, one may also be interested in obtaining the super-efficient index among efficient DMUs. The following model is to answer the call where it is a super two-stage SBM model with negative output:

$$\textbf{Min}\quad \phi_k^* = \frac{w_1 \times \left[1 + \frac{1}{m_1+m_2}\left(\sum_{i=1}^{m_1}\frac{t_i^p}{x_{ik}^p} + \sum_{i=1}^{m_2}\frac{t_i^n}{R_i^n}\left(1+\frac{x_{ik}^n}{\underline{x}_i^n}\right)\right)\right] + w_2}{w_1 + w_2 \times \left[1 - \frac{1}{s_1+s_2}\left(\sum_{r=1}^{s_1}\frac{t_r^p}{y_{rk}^p} + \sum_{r=1}^{s_2}\frac{t_r^n}{R_r^n}\left(1-\frac{y_{rk}^n}{\overline{y}_r^n}\right)\right)\right]}$$

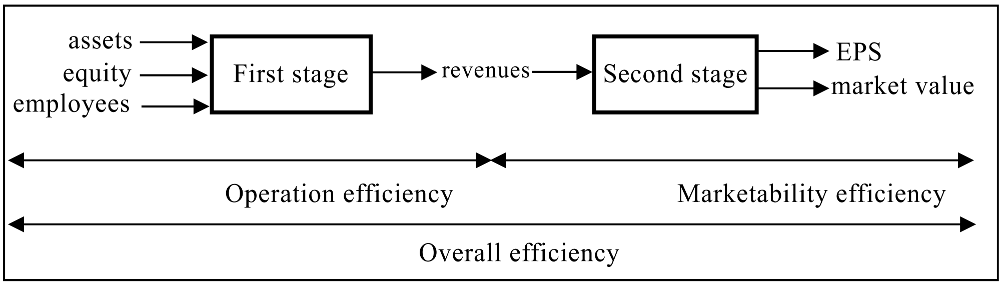
$$\begin{aligned}
\textbf{s.t.}\quad & x_{ik} + t_i^p \ge \sum_{j=1,\neq k}^{n} x_{ij}^p \lambda_j, \quad i=1,\ldots,m_1, \\
& x_{ik} + t_i^n \ge \sum_{j=1,\neq k}^{n} x_{ij}^n \lambda_j, \quad i=1,\ldots,m_2, \\
& \sum_{j=1,\neq k}^{n} \left(\lambda_j - \mu_j\right) z_{bj} = 0, \quad b=1,\ldots,l, \\
& y_{rk}^p - t_r^p \le \sum_{j=1,\neq k}^{n} y_{rj}^p \mu_j, \quad r=1,\ldots,s_1, \\
& y_{rk}^n - t_r^n \le \sum_{j=1,\neq k}^{n} y_{rj}^n \mu_j, \quad r=1,\ldots,s_2, \\
& \sum_{j=1,\neq k}^{n} \lambda_j = 1, \ \sum_{j=1,\neq k}^{n} \mu_j = 1 \quad j=1,\ldots,n, \\
& w_1 + w_2 = 1, \\
& \lambda_j, \mu_j, t_i^p, t_i^n, t_r^p, t_r^n \ge 0,
\end{aligned} \tag{5}$$

The logic is that the $\mathbf{DMU_k}$ is removed from the frontier and then recalculate the distance from the $\mathbf{DMU_k}$ to the one located on the new frontier (exclude $\mathbf{DMU_k}$). The super efficient index (ϕ_k^1) of the first stage is $1 + \frac{1}{m_1+m_2}\left(\sum_{i=1}^{m_1}\frac{t_i^p}{x_{ik}^p} + \sum_{i=1}^{m_2}\frac{t_i^n}{R_i^n}\left(1+\frac{x_{ik}^n}{\underline{x}_i^n}\right)\right)$, and the index ($\phi_k^2$) of the second stage is $\frac{1}{\left[1 - \frac{1}{s_1+s_2}\left(\sum_{r=1}^{s_1}\frac{t_r^p}{y_{rk}^p} + \sum_{r=1}^{s_2}\frac{t_r^n}{R_r^n}\left(1-\frac{y_{rk}^n}{\overline{y}_r^n}\right)\right)\right]}$. Accordingly, the greater the index (ϕ_k^*) is the higher of the rank of the DMU among efficient DMUs.

IV. Empirical Application

4.1 Description of Inputs, Intermediate Products and Outputs

To reveal more managerial information, we divide the production activity of commercial banks into operational and marketability processes. The division results into the inputs, intermediate products and outputs as a result enables organizations to investigate efficiency with respect to activities (Seiford and Zhu, 1999; Luo, 2003). The data for the empirical illustration were from the Taiwan Economic Journal database in 2009 with respect to the 24 banks (Taiwan Economic Journal Co. Ltd., 2010). The analysis framework was configured as Figure 1. The variables were defined as Table 1.



Source: Seiford and Zhu (1999).

Figure 1. Two-stage DEA model

Table 1. The definition and references of inputs and outputs

Dimension	Variables	References
Inputs	(x_1) Assets (unit: million NT$): sum of current assets, long-term assets, prepaid and deferred assets and intangible assets in a bank	Seiford and Zhu, 1999; Zha and Liang, 2010; Liu, Lu, and Lu, 2016
	(x_2) Equity (unit: million NT$): the value of an ownership interest in a bank in the form of common stock or preferred stock	Zha and Liang, 2010; Du *et al.*, 2011; Wanke and Barros, 2014
	(x_3) Employees (unit: person): the count of staffs in a bank	Seiford and Zhu, 1999; Zha and Liang, 2010; Du *et al.*, 2011; Liu, Lu, and Lu, 2016
Intermediate	(z_1) Operating revenues (unit: million NT$): the sales of goods and services by a bank and its subsidiaries	Seiford and Zhu, 1999; Zha and Liang, 2010; Du *et al.*, 2011
Outputs	(y_1) EPS (unit: percentage): the ratio of net profit to the outstanding shares	Du *et al.*, 2011; Liu, Lu, and Lu, 2016
	(y_2) Market value (unit: million NT$): the product of the weighted average stock price and the outstanding shares	Du *et al.*, 2011; Liu, Lu, and Lu, 2016

Source: This study.

Table 2 showed the descriptive statistics of the investigated 24 banks. To comply with the DEA modeling, it was necessary to test the isotonicity, therefore, the Pearson correlation analysis was conducted to check whether or not the inputs positively correlate the intermediate products and outputs. Table 3 showed the results and significantly confirmed the appropriateness for DEA modelling.

Table 2. Descriptive statistics regarding 24 Taiwan banks in 2009

Banks	No.	Assets	Equity	Employees	Revenues	EPS	Market value
Chang Hwa	1	1,475.327	81.017	6,519	29.361	0.50	946.945
First	2	1,921.430	89.913	7,094	37.039	0.42	966.910
Hua Nan	3	1,776.179	83.295	7,109	36.116	1.05	761.851
China Developemnt	4	236.937	135.305	696	11.017	0.76	760.519
Mega	5	2,212.313	156.250	5,722	50.218	1.66	1,186.033
Standard Chartered	6	616.131	35.009	4,208	22.305	-1.86	589.057
Taichung	7	309.362	15.361	1,878	7.370	0.01	124.691
Chinatrust	8	1,727.135	119.717	8,056	63.753	0.16	1,498.310
Cathay	9	1,527.967	95.458	6,551	37.216	1.66	3,120.939
Taipei Fubon	10	1,356.281	81.397	6,247	31.354	1.09	1,884.391
Taiwan business	11	1,187.559	41.727	5,150	22.680	0.37	321.896
Kaohsiung	12	186.111	9.434	924	3.813	0.15	54.946
Cosmos	13	136.449	15.954	1,640	7.945	-6.24	44.894
Union	14	360.221	19.259	3,289	12.048	0.13	122.805
SinoPac	15	1,057.097	62.360	4,965	26.881	0.47	578.278
E. Sun	16	934.797	51.507	4,433	19.904	0.56	425.772
Yuanta	17	365.054	23.653	2,344	8.047	0.21	489.464
Taishin	18	833.243	53.778	6,259	27.086	0.21	502.845
Far Eastern	19	377.052	20.763	2,155	9.985	0.64	232.056
TC	20	381.465	26.338	3,377	12.443	0.37	189.237
EnTie	21	303.725	18.351	1,679	7.738	0.04	155.538
Shin Kong	22	422.952	22.263	3,119	11.126	0.29	260.383
Jihsun	23	189.204	10.985	1,512	4.819	-9.09	37.970
Cooperative	24	2,589.351	111.007	9095	55.789	1.28	1,082.486
Min		136.449	9.434	696	3.813	-9.09	37.970
Average		936.806	57.504	4,330.042	23.169	-0.21	680.342
Max		2,589.351	156.250	9,095.000	63.753	1.66	3,120.939
Std. Dev.		724.588	42.489	2,388.288	16.471	2.37	697.355

Source: Arranged by study from Taiwan Economic Journal Co. Ltd. (2010).

Table 3. Correlation coefficients among inputs, intermediate products and outputs

Variable	x_1	x_2	x_3	z_1	y_1	y_2
(x_1) Assets	1.00	0.78 (0.00)	0.92 (0.00)	0.93 (0.00)	0.45 (0.00)	0.64 (0.00)
(x_2) Equity	-	1.00	0.63 (0.01)	0.81 (0.00)	0.45 (0.03)	0.68 (0.00)
(x_3) Employees	-	-	1.00	0.92 (0.00)	0.42 (0.04)	0.63 (0.00)
(z_1) Revenues	-	-	-	1.00	0.40 (0.05)	0.67 (0.00)
(y_1) EPS	-	-	-	-	1.00	0.40 (0.05)
(y_2) Market value	-	-	-	-	-	1.00

Source: This study.

Note: The numbers in parentheses represent the p-values.

4.2 Analysis of Operation and Marketability Efficiency

For modelling, bank's production data were divided into operation and marketability efficiencies, shown as the second, fourth and sixth columns in Table 4. To estimate those numbers, the equation (3) was applied by setting $w_1 = w_2 = 0.5$ and since negative inputs are unlikely to produce positive output in practice we could focus on positive inputs only. Notice that the overall efficiencies of bank 4 and 12 were both equal to unit. This may imply that these two banks suffered least losses during the 2008 global financial crisis. In addition, the banks that performed better in operation and average pure technical efficiencies achieved 0.8067 in operation efficiency and 0.503 in marketability. This showed that the marketability efficiency was the major source for inefficiency and might reflect investor's pessimistic concern of the financial market. Next, by applying the equation (5), the result showed that bank 4 was superior to bank 12 in terms of the overall, operation and marketability efficiency (shown in the third, fifth and seventh columns in Table 4). The bank 13 was only superior with a small margin in the first stage. That is to say, although bank 13 possessed good operational capability, it failed to gain profitability as well as the affirmation of general investors in the second stage. In the second stage, bank 9 was the most efficient bank, implying that this bank possessed better marketability and sales capacity.

Furthermore, after the arrival of efficiency scores with equation (3) for those in the frontier they would share the score in unit, for example the banks 4, 9 and 12 in Table 4 in the second stage. Given the results, by applying equation (5), bank 9 was shown to be superior to bank 4 and then bank 12.

Finally, the effectiveness of the handling negative output with the proposed model was illustrated as follows. For banks of negative EPS they were expected to attain poor performance (such as the banks 13 and 23 in Table 2). Table 4 showed that the overall efficiency with respect to bank 13 and 23 were 0.24 and 0.30, the lowest two among others.

Table 5 showed the mean efficiencies of the overall, operation and marketability efficiency scores in terms of bank asset size. In addition, since the investigated 24 banks were either in the financial holding format or the non-financial holding format, their mean efficiency scores were also shown in Table 5, respectively. The results revealed that large banks perform worse than small ones in terms of the operation efficiency, implying that large banks suffered greater losses during the 2008 financial crisis. Maybe it was because investors in general were more confident in large banks for their profitability and marketability capabilities, therefore, the marketability of large size banks were more appealing, even thought the small size banks were superior in overall and operation efficiencies.

Table 4. Overall, operation and marketability efficiency of banks in 2009

Banks No.	Overall		Operation		Marketability		Format*
	Score	Super	Score	Super	Score	Super	
1	0.5540	-	0.7648	-	0.4575	-	N
2	0.5272	-	0.6645	-	0.4636	-	F
3	0.4773	-	0.6985	-	0.3908	-	F
4	1.0000	1.3141	1.0000	1.6149	1.0000	1.0102	F
5	0.5640	-	0.5880	-	0.5508	-	F
6	0.6007	-	0.7884	-	0.5058	-	N
7	0.5147	-	0.7648	-	0.4117	-	N
8	0.6106	-	0.5904	-	0.6232	-	F
9	0.8561	-	0.7122	-	1.0000	1.2470	F
10	0.7554	-	0.5120	-	0.9984	-	F
11	0.3836	-	0.8231	-	0.2664	-	N
12	1.0000	1.2276	1.0000	1.4479	1.0000	1.0059	N
13	0.2432	-	1.0000	1.1213	0.1385	-	N
14	0.3970	-	0.7479	-	0.2939	-	N
15	0.4879	-	0.9534	-	0.3329	-	F
16	0.4505	-	0.9383	-	0.3028	-	F
17	0.7780	-	0.8666	-	0.7147	-	F
18	0.4714	-	0.9247	-	0.3243	-	F
19	0.5920	-	0.8784	-	0.4602	-	N
20	0.4273	-	0.9547	-	0.2797	-	N
21	0.5674	-	0.9141	-	0.4213	-	N
22	0.5600	-	0.8322	-	0.4402	-	F
23	0.2960	-	0.9249	-	0.1817	-	F
24	0.5159	-	0.5197	-	0.5140	-	N
Average	0.5679	-	0.8067	-	0.5030	-	-

Source: This study.

Note *: F=financial holding companies (FHCs), N= non-financial holding companies.

In terms of the banking formats, financial holding banks outperformed others at the overall and marketability efficiencies but worse at the operation efficiency. Nine of the thirteen banks in financial holding format belonged to the large size banks. It may imply that the large banks in financial holding format experiencing the worst performance during the 2008 financial crisis; it was the market confidence for future prospect that turned the overall efficiency to be greater for financial holding banks.

Table 5. Overall, Operation and marketability mean efficiency clustered by asset size and banking format

Grouping	Number of banks	Overall efficiency	Operation efficiency	Marketability efficiency
Assets size				
Large	12	0.5545	0.7241	0.5187
Small	12	0.5814	0.8893	0.4873
Format				
FHCs	13	0.6027	0.7851	0.5634
Non-FHCs	11	0.5269	0.8324	0.4317

Source: This study.

4.3 Projection Analysis

The merits of the proposed two-stage SBM models were not only taking negative outputs into account, but also providing projections regarding the appropriate amount of inputs, intermediate products and outputs. Therefore, projection analysis could provide an instrumental recommendation for inefficient banks to improve efficiency. Table 6 showed the application results of the equation (4). If inefficiency was attributed to the first stage, banks could decrease the amount of inputs to save resources. If inefficiency was attributed to the second stage, banks could increase the EPS and market value to appeal investors. For instance, the marketability efficiency of bank 9 attained the optimum level, there was no need of change in the amounts of intermediate products and outputs, however, the decrease of inputs could help to boost operational efficiency. On the other hand, bank 13 has attained operational efficiency, hence the amounts of inputs and intermediate products needed no change; however, its output resources could be increased to attain marketability efficiency.

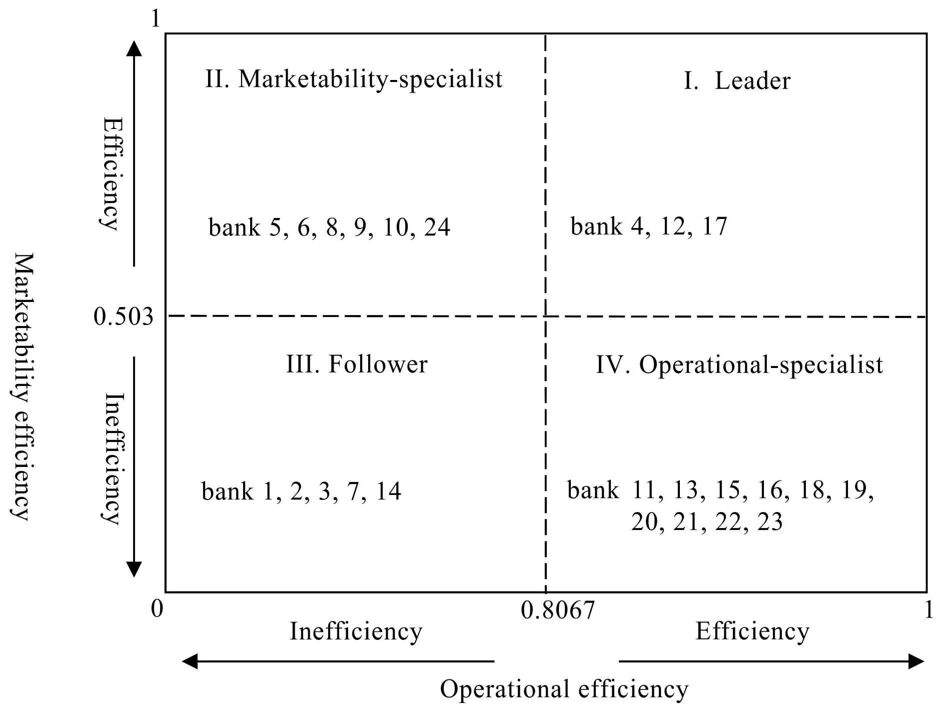
Table 6. Projection analysis of inputs, intermediate products and outputs

Banks No.	Assets (million NT$)	Equity (million NT$)	Employees (person)	Revenues (million NT$)	EPS (%)	Market value (million NT$)
1	970.755	70.377	5,005	37.216	1.66	3,120.939
2	970.756	70.377	5,005	37.216	1.66	3,120.939
3	970.756	70.377	5,005	37.216	1.66	3,120.939
4	236.937	135.305	696	11.017	0.76	760.519
5	970.755	70.377	5,005	37.216	1.66	3,120.939
6	428.562	35.009	2,818	18.193	1.01	1,407.097
7	140.965	15.361	1,575	7.569	0.20	440.428
8	970.755	70.377	5,005	37.216	1.66	3,120.939
9	970.755	70.377	5,005	37.216	1.66	3,120.939
10	579.559	44.859	3,427	23.491	1.19	1,884.391
11	704.242	41.727	4,513	25.592	1.26	2,073.679
12	186.111	9.434	924	3.813	0.15	54.946
13	136.449	15.954	1,640	7.945	0.21	478.996
14	219.648	19.259	2,085	10.435	0.65	707.464
15	909.338	62.360	4,965	34.252	1.56	2,853.876
16	761.732	51.507	4,433	28.826	1.37	2,365.038
17	254.475	23.653	2,116	12.086	0.80	856.817
18	833.243	53.778	4,845	30.852	1.44	2,547.556
19	239.545	20.763	2,155	11.175	0.77	774.785
20	381.465	26.338	2,918	15.411	0.91	1,156.426
21	225.425	18.351	1,679	9.016	0.39	577.892
22	295.269	22.263	2,490	12.699	0.82	912.106
23	189.204	10.985	1,171	4.916	0.17	168.163
24	970.755	70.378	5,005	37.216	1.66	3,120.939

Source: This study.

4.4 Managerial Fitness Matrix

The success of a bank is due not only on the operational efficiency but also the marketability efficiency, hence the performance information solely about inputs and outputs is not up to this demand; the intermediate production information is a prerequisite. To this call, this paper proposed a matrix which helps to configure the projection analysis results as follows. Figure 2 for instance used the operational and marketability efficiency as the horizontal and vertical axis, respectively. Dividing on the mean values of operational efficiency (0.8067) and marketability efficiency (0.503), four quadrants of banks were derived. For the ease of discussion, the four quadrants were noted as I-Leader, II-Marketability-specialist, III-Follower and IV-Operational-specialist.



Source: This study.

Figure 2. Managerial fitness matrix

Quadrant I – the Leader. There were three banks (4, 12 and 17) in this quadrant that were both doing well in operational and marketability. Amongst, bank 4 and 12 were the best and following by bank 17, therefore, constant maintaining of competitive advantages was advised for banks 4 and 12, whereas bank 17 was advised to advance her market strategy to balance her operational superiority.

Quadrant II – the Marketability-specialist. The banks (5, 6, 8, 9, 10 and 24) in this quadrant displayed good marketability efficiency however their operation efficiency was below average. Obviously, these banks needed to advance their operation efficiency. Benchmarking on the leaders in quadrant I these banks were advised to reduce inputs or expand operation.

Quadrant III – the Follower. Five banks (1, 2, 3, 7 and 14) were in this quadrant. Both of their operation efficiency and marketability efficiency were below the average. In other words, improvement in operation or marketability was appropriate for banks in this quadrant. Targeting to move into quadrant II or IV was eligible.

Quadrant IV -- Operational-specialist. These banks (11, 13, 15, 16, 18, 19, 20, 21, 22 and 23) possessed relatively good operational efficiency but needed to improve their marketability performance. These banks made up approximately 42% of total samples and it may hint the difficulty for the banks in Taiwan to advance in the marketability area. Thus, an adjustment of the marketing strategy to increase investors' confidence was advised.

V. Conclusions and Remarks

The 2008 financial crisis reminded the necessity of DEA modelling with negative outputs. This paper modifies the objective function of the SBM and two-stage SBM models to handle negative outputs and the separation of operation and marketability efficiencies. In addition, a super two-stage SBM model, an extension of the proposed two-stage SBM model, was introduced to handle the oftentimes encountered data format, the ranking scores. In addition, the model can be applied in subsequent the energy, environment, manufacturing related studies due to it doesn't need to impose any assumptions of functional form on production function.

The application of the two-stage SBM model revealed that the banks in Taiwan generally performed better in operation capacity but less effective in marketability and sales. This finding coincided with Luo (2003). Furthermore, small banks tended to outperform large ones in operations but lose in marketability. Meanwhile, financial holding banks were superior to non-financial holding banks in marketability and overall efficiencies. Finally, the separation on operation and marketability efficiencies and the inclusion of intermediate production information endeavored further projection analysis. This analysis results may be configured in a matrix to better reveal managerial implications.

Nevertheless, the paper is subject to some limits. Firstly, the study assumes that the intermediate products need be positive in the two-stage SBM model with negative output. Secondly, the efficiency analysis adopted in this paper is measured from a static point of view within a period. However, the change of the efficiency over different periods is likely associated with the references when management decision was made. Thirdly, the featured two-stage DEA assumes that the output in the first stage is equal to the input of the second stage. In practice, the input of the second stage is not necessarily the output in the first stage. Finally, DEA models oftentimes deal with undesirable data (Cooper, Seiford, and Tone, 2007; Fukuyama and Weber, 2010; Liu *et al.,* 2010). As a result, the categorization of the undesirable data and the alignment of data categories with the appropriate DEA models are prerequisites for accurate efficiency assessments.

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考量負值產出與超效率之二階段差額變數基礎模型：以台灣商業銀行爲例

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雖然組織的負面績效並不常見，但最近的資料包絡分析文獻卻開始在探討要如何來衡量其績效。然而，這些文獻目前卻只探討到一階段的績效評估，往往忽略了中間生產過程的資訊，故本研究發展出兩階段差額變數基礎衡量（SBM）模型和超效率兩階段差額變數基礎衡量模型來處理負值產出、中間生產資訊及效率值的排名。實證研究方面，本文利用台灣 24 家商業銀行的財務資料來驗證上述所推導出來的模型，研究結果發現規模較大的商業銀行，其經營效率的表現反而比規模較小的商業銀行差，但其在市場效率的表現上，卻是優於規模較小的商業銀行。另外，金融控股體制下的商業銀行，其整體效率（經營和市場方面）則優於傳統商業銀行（非金控體制下）。最後本文藉由管理矩陣，提供相對無效率銀行的管理者更多管理方向及改善效率之資訊。

關鍵詞：網路資料包絡分析、績效評估、黑箱、矩陣管理

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