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解析勞動儲藏與勞動報酬短期遞增的關係

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Labor Hoarding, Dynamic Adjustment and Returns to Labor

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Literature on the short-run relationship between output and employment has focused on the empirical phenomenon called "short-run increasing returns to labor" (SRIRL). SRIRL signifies the phenomenon that fluctuations in output induce a variation of labor input less than proportional. The empirical finding of SRIRL represents a clear contradiction to the traditional theory of firm. There have been attempts to resolve this paradox, one of the general explanations offered involved the concept of labor hoarding. This research attempts to examine the role of labor hoarding in SRIRL by applying a dynamic dual model of explicit cost minimization. As an alternative approach to resolving the SRIRL paradox, the dynamic dual model offers greater flexibility in the specification of production technology. The results suggest that SRIRL does not necessarily contradict intertemporal optimization behavior and labor hoarding is neither sufficient nor necessary for SRIRL.

Keywords:*Labor hoarding; Dynamic adjustment; Returns to labor*

1. Introduction

Literature on the short-run relationship between output and employment

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has focused on the empirical phenomenon called "short-run increasing returns to labor" (SRIRL). SRIRL signifies the phenomenon that fluctuations in output induce a variation of labor input less than proportional. The major implication of SRIRL is the systematic cyclic variation in labor productivity; that is, although labor productivity diminishes during a cyclic decline, it increases rapidly from a trough. The empirical evidence provided by early studies such as Hultgren (1960), Kendrick (1961) and Kuh (1965) support such procyclic behavior of man-hour productivity.

The empirical finding of SRIRL represents a clear contradiction to the traditional theory of firm. There have been attempts to resolve this paradox and to link the empirical finding to the underlying theory. One of the general explanations offered involved the concept of labor hoarding. Solow (1968) formalized the labor hoarding concept by emphasizing the adjustment cost associated with rapid variation of the level of employment, and concluded that in firms' attempts to minimize the cost of production, they retain labor even in periods of decline. Therefore, according to the labor hoarding hypothesis, the procyclic pattern of labor productivity follows directly from behavior of intertemporal cost minimization.

Not until recent development in empirically implementable models of interrelated factor demands, has there been empirical research based on a model of labor hoarding of the Solow-type. Using the classic calculus of variations approach, Morrison and Berndt (1981) examine the role of labor hoarding in SRIRL. Because of the difficulties in obtaining a closed-form analytic expression of the equilibrium, the model proposed by Morrison and Berndt is restricted to linear-quadratic technologies. This research attempts to examine the role of labor hoarding in SRIRL by applying a dynamic dual model of explicit cost minimization. As an alternative approach to resolving the SRIRL paradox, the dynamic dual model offers greater flexibility in the specification of production technology.

Explicitly defining SRIRL as the case in which the proportional growth of labor is smaller than that of output in the short-run, and when growth of labor in the short-run is smaller than that in the long-run, this paper demonstrates that SRIRL does not necessarily contradict intertemporal optimization behavior and labor hoarding is neither sufficient nor necessary for SRIRL.

The remainder of the paper is organized as follows. In the next section, the dynamic dual model of explicit cost minimization is presented. Propositions summarizing the role of labor hoarding in SRIRL are proposed. Section 3 gives a brief concluding remarks.

2. SRIRL in the Dynamic Dual Model of Production

The firm's production technology is described by the single-output production function $Y = F(X, K, \dot{K}, \tau)$, which possesses all standard properties of the neoclassical theory of production. The production function is single valued, defining the maximum output obtainable from a specified set of inputs. It is a positive, continuous, twice-differentiable function with positive marginal product from variable inputs, $(X_1, X_2, \cdots, X_n)$, and from quasi-fixed inputs, $(K_1, K_2, \cdots, K_m)$. The argument τ represents a proxy for the advancement of technology and the inclusion of net investment, $\dot{K}$, in the production function reflects the internal cost associated with adjusting quasi-fixed factors in terms of foregone output.

The adjustment cost is internal in the sense that expanding (contracting) the quasi-fixed factor stocks will result in a decrease (increase) in output.

Therefore, the product of $\dot{K}$ and $F_{\dot{K}}$ is always negative. In addition, to assure the sluggish or gradual behavior in adjusting the levels of quasi-fixed factors, the diseconomies accompanying adjustment is assumed to be greater the faster the adjustment takes place. This assumption is equivalent to the convexity assumption of the adjustment cost function.

The optimization problem for the firm that seeks to minimize the discounted stream of costs is stated as

$$\begin{aligned}
&\underset{\dot{K}(\tau),X(\tau)}{\text{minimize}} \int_t^\infty e^{-r(\tau-t)}[W(\tau)'X(\tau)+C'(\tau)K(\tau)]dt,\\
&\text{subject to } \dot{K}(t)=I(t)-\delta K(t), K(t)=k\\
&\qquad Y(\tau)=F(X(\tau),K(\tau),\dot{K}(\tau),\tau), \forall 0\le\tau<\infty \qquad (1)
\end{aligned}$$

in which $W(\tau)$ and $C(\tau)$ denote, respectively, the price vectors of variable and quasi-fixed inputs at time τ. The constant discount rate is denoted by r. Let the optimal value function giving the minimum value of the problem realizable from some time t on be denoted by $J(Y,w,c,k,t)$, Epstein (1981) demonstrated that the value function and underlying production technology is theoretically obtainable from one the other by solving the appropriate static optimization problem as expressed in the Hamilton-Jacobi equation,

$$\begin{aligned}
rJ(Y,w,c,k,t) = &\underset{\dot{K}(t),X(t),\lambda(t)}{\text{minimize}} \{w'X+c'k+(I-\delta k)'J_k(.)+\lambda(Y(t))\\
&-F(X(t),K(t),\dot{K}(t),t)\}+J_t(.). \qquad (2)
\end{aligned}$$

Here λ is the Lagrangian multiplier associated with the production technology constraint, and w and c denote, respectively, current prices for variable and quasi-fixed inputs.

The discussion of SRIRL centers on the relationship between proportional growth of output and labor inputs. To establish this relationship within the dynamic adjustment framework, we start with the decomposition of output growth under intertemporal cost minimization which was initially formulated by Stefanou (1988). The proportional growth of output is decomposed as

$$\hat{Y}(t) = \sum_{i=1}^{n} \frac{w_i X_i^*}{\lambda^* Y(t)} \hat{X}_i + \sum_{j=1}^{m} \frac{J_{k_j}(.)\dot{K}_J^*}{\lambda^* Y(t)} \hat{\dot{K}}_j + \sum_{j=1}^{m} \frac{\left(-rJ_{k_j}(.)+c_j+\frac{dJ_{k_j}(.)}{dt}\right) k_j}{\lambda^* Y(t)} \hat{K}_j + \frac{\partial F(X,k,\dot{K},t)}{\partial t} \frac{1}{F(X,k,\dot{K},t)}. \quad (3)$$

Explicitly define SRIRL as the case in which the proportional growth of labor is smaller than that of output in the short-run, and when growth of labor in the short-run is smaller than that in the long-run. The following propositions concluded that SRIRL is not necessarily a paradox in the dynamic adjustment cost models.

Proposition 1. *Given that labor is variable in nature, SRIRL will be present when the followings hold:*

(i) *the proportional growth of labor, $\hat{L}$, is smaller than the weighted average proportional growth of other variable inputs $\sum_{i=2}^{n} S_i \hat{X}_i$, and*

(ii) *the weighted average proportional growth of quasi-fixed factor stocks is negative in the long-run.*

<Proof> To illustrate that SRIRL may be present even when labor input is adjusted freely, it is shown below that under some conditions the proportional growth of labor is smaller than that of output in the short-run and that the growth of labor in the short-run is smaller than its long-run counterpart.

In the short-run, $\dot{K} = \hat{K} = 0$ since quasi-fixed factors are fixed. Let labor be the first variable input, and the component of technical change be denoted by $\hat{A}$, (3) can be alternatively expressed as

$$\hat{Y}(t) = S_1\hat{L} + \sum_{i=2}^{n} S_i\hat{X}_i + \hat{A}.$$

in which $S_i = (w_i X_i^*)/(\lambda^* Y(t))$. Subtracting the short-run proportional growth of output from that of labor yields

$$(\hat{L} - \hat{Y})^{SR} = \left((1 - S_1)\hat{L} - \sum_{i=2}^{n} S_i\hat{X}_i\right) - \hat{A}. \tag{4}$$

One of the conditions for SRIRL to occur is that $\hat{L} - \hat{Y} < 0$ in the short-run. Because the component of technical change is in general positive, $\hat{L}$ minus $\hat{Y}$ is negative as long as $(1 - S_1)\hat{L} - \sum_{i=2}^{n} S_i\hat{X}_i < 0$. Given that $(1 - S_1)\hat{L} < \hat{L}$, (4) is negative as $\hat{L} < \sum_{i=2}^{n} S_i\hat{X}_i$. Therefore, as the proportional growth of labor is smaller than the weighted average proportional growth of other variable inputs, $\sum_{i=2}^{n} S_i\hat{X}_i$ proportional growth of labor is smaller than that of output in the short-run.

To compare the growth of labor and that of output in the long-run, note that in the steady state, $\dot{K} = \dot{J}_k(.) = 0$. Therefore, as each input is at its respective long-run equilibrium level, (3) becomes

$$\hat{Y}(t) = \sum_{i=1}^{n} \frac{w_i X_i^*}{\lambda^* Y(t)}\hat{X}_i + \sum_{j=1}^{m} \frac{(-rJ_{k_j}(.) + c_j)k_j}{\lambda^* Y(t)}\hat{K}_j.$$

In the long run, subtracting the proportional growth of output from that of labor leads to

$$(\hat{L} - \hat{Y})^{LR} = \left((1 - S_1)\hat{L} - \sum_{i=2}^{n} S_i\hat{X}_i\right) - \hat{A} - \sum_{j=1}^{m} V_j\hat{K}_j, \tag{5}$$

in which $V_j = [(-rJ_{k_j}(.)+c_j)k_j]/[\lambda^* Y(t)]$. From (4) and (5), as $\sum_{j=1}^{m} V_j\hat{K}_j < 0$ the proportional increase in labor is clearly smaller in the short-run than in the long-run. Therefore, the second condition for SRIRL to occur is that the weighted average proportional growth of quasi-fixed stocks is negative in the long-run.

Proposition 1 states that SRIRL may be present even when labor input is adjusted freely. This condition in turn suggests that the presence of SRIRL in the dynamic adjustment cost model does not require labor hoarding, but rather relies on the relationship between proportional growth of labor and other inputs. Consequently, labor hoarding is not necessary for SRIRL.

Proposition 2. *As adjustment of labor is costly, i.e., as labor hoarding exists, SRIRL may not occur when the following conditions hold:*

(i) *production technology is stationary and exhibits long-run constant returns to scale;*

(ii) *the opportunity cost of an additional unit of quasi-fixed factor is grater than the instantaneous capital gain associated with acquisition of the additional unit of capital; and*

(iii) *the adjusted growth of labor,* $\hat{L}\sum_{i=1}^{n} S_i$ *and* $\hat{L}\sum_{j=2} U_j$ *are greater than, respectively, the weighted average growths of variable inputs* $\sum_{i=1}^{n} S_i\hat{X}_i$, *and of other quasi-fixed inputs,* $\sum_{j=2}^{n} U_j\hat{K}_j$.

<Proof> When labor is quasi-fixed in nature, the definition of lengths of runs has to be revised. The reason is that if quasi-fixed inputs are assumed to be fixed in the short run, then SRIRL is definitely present since in the short run $\hat{L} - \hat{Y} = -\hat{Y} < 0$ and $(\hat{L} - \hat{Y})^{SR}$ is invariably less than its long-run counterpart. Therefore, in what follows, the short-run is redefined as the time span allowing partial adjustment of stock variables, and long-run is the time span in which quasi-fixed inputs have adjusted fully to their respective

long-run equilibrium levels.

It can be shown that as production technology is stationary and exhibits long-run constant returns to scale, optimality requires that the shadow value of output equals instantaneous cost flow, that is,

$$\lambda^* Y(t) = w'X^* + c'k. \tag{6}$$

To see this, note that stationary technology implies that $\partial F/\partial t = 0$. The dynamic measure of cost elasticity is the ratio of long-run marginal cost over long-run average cost (Stefanou, 1989), $LRMC/LRAC = YJ_y(.)/J(.)$. Since $J_y(.) = [\lambda^* + (dJ_y(.)/dt)]/r$ and $dJ_y(.)/dt = dJ(.)/dt = 0$ in the long-run, constant returns to scale implies

$$Y\left(\frac{\lambda^*}{r}\right) = \frac{w'X^* + c'k}{r},$$

given that technology is stationary. Let labor be the first quasi-fixed input; defining $S_i = w_i X_i^*/\lambda^* Y(t), U_j = c_j k_j/\lambda^* Y(t)$, subtracting the proportional growth of output from that of labor, we have

$$(\hat{L} - \hat{Y})^{SR} = \hat{L} - \sum_{i=1}^{n} S_i \hat{X}_i - U_1 \hat{L} - \sum_{j=2}^{m} U_j \hat{K}_j - \sum_{j=1}^{m} \frac{J_{k_j}(.)\hat{K}_j}{\lambda^* Y(t)} \hat{\dot{K}}_j$$
$$- \sum_{j=1}^{m} \frac{\left(-rJ_{k_j}(.) + \dfrac{dJ_{k_j}(.)}{dt}\right) k_j}{\lambda^* Y(t)} \hat{K}_j.$$

Using the relationship that $U_1 = 1 - \sum_{i=1}^{n} S_i - \sum_{j=2}^{m} U_j$ and rearranging the terms yield

$$(\hat{L} - \hat{Y})^{SR} = \sum_{i=1}^{n} S_i(\hat{L} - \hat{X}_i) + \sum_{j=2}^{m} U_j(\hat{L} - \hat{K}_j) + \sum_{j=1}^{m} \frac{-J_{k_j}(.)\dot{K}_j}{\lambda^* Y(t)} \hat{\dot{K}}_j$$

$$+\sum_{j=1}^{m}\frac{\left(rJ_{k_j}(.)-\dfrac{dJ_{k_j}(.)}{dt}\right)k_j}{\lambda^*Y(t)}\hat{K}_j. \tag{7}$$

SRIRL is not plausible when fluctuations in output induce more than proportional fluctuations of labor, in this case $(\hat{L}-\hat{Y})^{SR}>0$. Referring back to the first-order condition of long-run cost minimization, $J_{k_j}(.)=\lambda F_{\dot{K}_j}$, the third term in (7) is positive because expanding the capacity of capital ($\dot{K}>0$) results in a positive adjustment cost in terms of foregone output and $F_{\dot{K}}<0$. Consequently, as long as $\hat{L}\sum_{i=1}^{n}S_i>\sum_{i=1}^{n}S_i\hat{X}_i, \hat{L}\sum_{j=2}^{m}U_j>\sum_{j=2}^{m}U_j\hat{K}_j$ and $rJ_k(.)>dJ_k(.)/dt, (\hat{L}-\hat{Y})^{SR}$ remains positive. That is, proportional growth of output induces more than proportional growth of labor input even in the presence of labor hoarding when (i) opportunity cost of an additional unit of quasi-fixed factors exceeds the instantaneous capital gain associated with acquisition of the additional unit of capital; and (ii) the adjusted growth of labor exceeds, respectively, the weighted average growths of variable and of other quasi-fixed inputs.

3. Concluding Remarks

Literature on the short-run relationship between output and employment has focused on the empirical phenomenon called "short-run increasing returns to labor" (SRIRL). SRIRL signifies the phenomenon that fluctuations in output induce a variation of labor input less than proportional. The empirical finding of SRIRL represents a clear contradiction to the traditional theory of firm. There have been attempts to resolve this paradox, one of the general explanations offered involved the concept of labor hoarding. This research attempts to examine the role of labor hoarding in SRIRL by applying a dynamic dual model of explicit cost minimization. As an alternative approach to resolving the SRIRL paradox, the dynamic dual model offers greater flexibility in the specification of production technology.

Propositions 1 and 2 concluded that SRIRL is not necessarily a paradox in the dynamic adjustment cost models and that labor hoarding is neither sufficient nor necessary for SRIRL. As shown in Proposition 2, when there exist costs associated with adjusting labor, i.e., when labor hoarding is a rational behavior for long-run cost minimizing firms, SRIRL may not be present. Since labor hoarding does not imply SRIRL, labor hoarding is not sufficient for SRIRL. Furthermore, according to Proposition 1, observing SRIRL does not require labor hoarding, but rather relies on the relationship between proportional growth of labor and other inputs. Since labor hoarding is not necessarily true when SRIRL is observed, labor hoarding is not necessary for SRIRL. Since the results indicate that SRIRL is not necessarily a paradoxical observation for the intertemporally optimizing firm facing adjustment costs, it ought to be treated as an open question for empirically oriented research.

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解析勞動儲藏與勞動報酬短期遞增的關係

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過去在相關文獻中，勞動報酬的短期遞增是一極重要的實證發現。該種現象是指，在短期時勞動的需求產出彈性，亦即每百分之一的產量變動造成勞動需求的百分比變動，會小於一，而在長期時，勞動需求產出彈性會較短期的值來得大。勞動報酬短期遞增的發現很明顯的與傳統的廠商理論相矛盾，文獻中曾有些學者嚐試提供一些解釋，其中最著名的是 Solow(1968) 所提出的勞動儲藏理論。Solow 以迅速調整勞動僱用所產生的調整成本來說明勞動儲藏的理性行為，並由此解釋勞動報酬的短期遞增現象。Solow 之後，Morrison and Berndt(1981) 嚐試以傳統變分法來解析勞動儲藏與短期勞動報酬遞增的關係。由於尤拉方程式求解的困難，Morrison and Berndt 的模型必須對生產技術做極大的限制，因此，本研究以動態對偶模型來探討勞動儲藏與勞動報酬短期遞增之間的關係。在廠商追求跨期成本現值總和極小的行為假設下，藉著將產出成長分解的結果，我們證明勞動儲藏既非勞動報酬短期遞增的充分條件亦非必要條件。這個結果說明在調整成本的模型中，勞動報酬的短期遞增既非一種矛盾亦非如 Solow 所言為勞動儲藏的必然結果，因此，勞動報酬的短期遞增純粹是一個實證的議題。

關鍵詞：勞動儲藏，調整成本，短期勞動報酬遞增

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