A MODEL OF CONTRACTING AND HEDGING: THEORY AND APPLICATION TO THE HOG INDUSTRY

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The paper analyzes the farmer's and the meat packer's decisions when both forward contracting and hedging with live-hog futures can be used in risk management. The optimal contracting ratio and the optimal hedge ratio are derived. A numerical example for the U.S. feeder-to-finish enterprise is used to demonstrate the application of the optimal rules. The "Contracting-and-hedging" strategy is compared with the "Only-contracting," "Only-hedging," and "No risk management" strategies using the criterion of expected utility. The results show that contracting and hedging are substitutes for farmers, while they are complements for packers.

Keywords: *forward contracting, hedging, risk, feeder-to-finish enterprises*

1. INTRODUCTION

Since the 1980s the hog industry has accelerated toward vertical coor-

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dination or agricultural industrialization (Shapouri et al., 1994; Hurt, 1994). One industry trend is that market participants have increased the use of forward contracts and futures contracts to protect themselves against adverse price movements. How to use these instruments properly becomes a critical issue of vertical coordination for this industry. However, the traditional optimal-hedge-ratio is not appropriate in this case because when a farmer or a meat packer enters a forward contract his probability density function (p.d.f.) of prices is changed due to the price rigidity of the contract. The structure of returns and risk, which produces the optimal hedge ratio, will change when the p.d.f. changes. The traditional hedge ratio will be over- or under-estimated because the effect of contracting is ignored. It is more appropriate to solve for the contracting ratio and the hedge ratio simultaneously using a model that considers contract and futures transactions together. Accordingly, the objective of this paper is to model the interaction between forward contracting and hedging with futures, and then to derive the optimal strategies of using forward and futures contracts.

2. PREVIOUS RESEARCH

A (cash) forward contract is a private agreement between the participants in the hog industry. Farmers, feedmills, meat packers, and marketing cooperatives are active users of forward contacts. Many variations in forward contracts are used in this industry (Dobson et al., 1988; Gillespie & Eidman, 1933): for example, forward sale contracts for slaughter hogs, marketing agreements for feeder pigs, custom-feeding contracts, and breeding stock leases. This discussion focuses on the forward sale contract for slaughter hogs, which will be referred to as the "forward contract". A futures contract is a standardized and publicly accepted forward contract traded in a futures market. Live-hog, pork bellies, corn, and soybean meal futures contracts are important to the hog industry, but only live-hog futures are discussed here. Therefore, "Hedging" in this paper, means hedging with the live-hog futures contract.

Theoretically speaking, the existence of contracts yields a multi-price system. There are at least two prices prevailing in the market: the spot price and the contract price (Polinsky, 1986). In our case, the futures price is a third. The spot price traditionally is the market-clearing price level in the Walrasian framework, and it always adjusts to the contemporary demand and supply fundamentals. In contrast, the contract price has price rigidity during the contract life. Therefore, contracts, risk, price rigidity, and market equilibrium are always discussed together. In two pioneer articles to uncover the relation of contracting and price rigidity, Carlton (1978, 1979) investigated a market characterized by uncertainty and transaction costs. He showed that uncertainty and transaction costs created incentives for firms to use forward contracts. Therefore, rigid prices and delivery lags were not necessarily disequilibrium phenomena. An empirical study of Hubbard and Weiner (1989) showed that contracting affected copper and oil price adjustments. Moreover, the effect would in turn influence the effectiveness of any stockpile policy for price stabilization. The impact of contracting was not only found on non-agricultural prices, but also on agricultural prices. Schroeder et al. (1993), for example, used a data set including a contract basis from six Texas feedlots to show that as another outlet of products contracting had a negative influence on average spot price.

Hubbard and Weiner (1991) suggested that facing the multi-price system, an agent's considerations will be two-sided risk aversions, expected revenues, and expected risks associated with spot and contracting market systems. Haydu et al. (1992) investigated farmer's incentive to forward purchase inputs. They derived an optimal contracting rule from their model. Results showed that the primary reasons farmers contracted inputs were to reduce risk and to speculate on favorable price moves. A numerical example of fertilizer used in corn production indicated that the size of the price discount was the dominant factor.

As mentioned above, contracting and hedging are closely related marketing instruments. It is natural to compare them. There has been voluminous study about the relationship between forward contract prices and futures prices in financial economics (Cornell and Reinganum,1981; Jarrow and Oldfield, 1981; Cox et al., 1981). A typical view is that arbitrage keeps a stable relationship between forward prices and futures prices, and the differences are attributed to taxes, risk free interest rates, transaction costs, and the settlement procedure for the futures market. However, financial commodities are very different from agricultural commodities in many aspects. For example, the main holding costs of financial commodities are interest and tax costs, while those of agricultural commodities are storage costs (for storable commodities such as corn) or feedlot service costs (for unstorable commodities such as live hogs), and transportation costs (Paul and Wesson, 1967). These essential differences make financial economic research less applicable.

Elam (1992) examined forward contracting of fed cattle. Compared to a future contract, a forward contract removes spot price risk. However, the contract price was estimated to be lower than the futures hedge price by .28−.59/cwt. for steers and .86−1.64/cwt. for heifers. This result, as well as Haydu et al. (1992) and Schroeder et al. (1993) will be arguments for concern about a nonzero risk premium which is contrast to the assumption of Hubbard and Weiner (1989). Hubbard and Weiner assumed that the trading parties are risk-neutral, the contract price equal the expected spot price, and the risk premium is zero. Elam's work revealed the significant difference between forward contract prices and futures prices and suggested the important role of risk premium in forward pricing.

The above research presented a theoretical and empirical foundation about 1) the relation between contracting and price rigidity, 2) the factors influencing contracting behavior, 3) the discrepancy between forward contract prices and futures prices. As a further research, the paper provides a model allowing forward contracts and futures as means of coping with price risk to study an economic agent's optimal strategy. Specifically, this paper considers the simultaneous choices of contracting and hedging within the mean-variance framework from which the optimal contracting ratios and hedge ratios are derived.

3. MODEL

A competitive market structure is assumed for simplicity. This allows us to focus on the role of hedging in modeling the farmer's and packer's problems. It means that both farmers and packers are price-takers who view prices as given, and choose quantities of hogs to maximize their expected utility. Although this example uses hogs, it can be applied to other industries after some modification.

3.1 The Farmer's Problem

In the production sector of hogs, there are many homogeneous farmers raising hogs for sale. The hogs will be sold when they reach market weight, 240 lb. Then, farmers must choose marketing instruments to manage the price risk. Suppose that farmers can use a forward contract or futures to improve their risk-management, such that a representative farmer has a profit function $\prod^F$ as follows:

$$\prod^F = (1-\alpha^F)p_s q^F + \alpha^F p_c q^F - C(q^F) + \beta^F q^F (f - f_0) \tag{1}$$

where α^F = the farmer's contracting ratio, p_s = the spot price for hogs, q^F = the farmer's total number of hogs produced, p_c = the contract price for hogs, $C(\cdot)$ = the cost function of hog production with $C' > 0$ and $C'' > 0$, [1] β^F = the farmer's hedge ratio with negative numbers denoting short positions, f_0 = the futures price at decision time, and f = the futures price at delivery time.

Without losing much generality, let $q^F = 1$ to simplify the model deduction. Accordingly, (1) can be rewritten as

$$\prod^F = (1-\alpha^F)p_s + \alpha^F p_c - C + \beta^F (f - f_0) \tag{2}$$

There are three reasons to justify the normalization of quantities. (i) The reason of a partial equilibrium analysis---if the focus is the marketing decision and the production plan is given, then the quantity of hogs produced is fixed and can be treated as a lump-sum unit i.e., equal to one. This assumption is like the case discussed in Kahl (1983) where the cash position is given. (ii) An average-profit point of view. (iii) A point of full utilization of productivity capacity--if assume that farmers and packers fully utilize their productivity capacity just as Carlton (1979) did, then the quantity will be fixed and inelastic to price changes.

The mean and variance of the farmer's profit are

$$E(\prod^F) = (1-\alpha^F)\overline{p}_s + \alpha^F p_c - C + \beta^F (\overline{f} - f_0) \tag{3}$$

$$Var(\prod^F) = (1-\alpha^F)^2 \sigma_s^2 + (\beta^F)^2 \sigma_f^2 + 2(1-\alpha^F)\beta^F \sigma_{sf} \tag{4}$$

where $\overline{p}_s$ = the expected spot price at delivery, $\overline{f}$ = the expected futures price at delivery, σ_s^2 = the variance of the spot price, σ_f^2 = the variance of the futures price, and σ_{sf} = the covariance of spot and futures prices. Under a mean-variance framework, the farmer's problem is to maximize expected utility Φ^F by choosing the contracting ratio α^F and the hedge ratio β^F; that is,

$$\underset{\alpha^F \beta^F}{Max}\, \Phi^F = E(\Pi^F) - \frac{\lambda}{2} Var(\Pi^F)$$

The first-order conditions require

$$\overline{p}_s - p_c = \lambda \left[(1-\alpha^F)\sigma_s^2 + \beta^F \sigma_{sf}\right] \tag{5}$$

$$\overline{f} - f_0 = \lambda \left[\beta^F \sigma_f^2 + (1-\alpha^F)\sigma_{sf}\right] \tag{6}$$

(5) shows that the farmer should choose α^F such that the risk premium $\overline{p}_s - p_c$ paid equals the marginal reduction in risk by contracting, $(1-\alpha^F)\sigma_s^2 + \beta^F\sigma_{sf}$, adjusted by the coefficient of risk aversion λ. (6) shows that the farmer should choose β^F such that the expected hedging profit (loss) $\overline{f} - f_0$ equals the marginal change in risk by hedging $\beta^F\sigma_f^2 + (1-\alpha^F)\sigma_{sf}$, adjusted by λ. To ensure that the maximum is reached through the first-order conditions requires the second-order sufficient conditions. The second-order conditions are shown to be satisfied in the appendix. Moreover, solving the first-order conditions for (α^F, β^F) gives

$$\alpha_{CH}^F = \frac{-\sigma_f^2\left(\frac{\overline{p}_s - p_c}{\lambda} - \sigma_s^2\right) + \sigma_{sf}\left(\frac{\overline{f} - f_0}{\lambda} - \sigma_{sf}\right)}{\sigma_s^2\sigma_f^2 - \sigma_{sf}^2} \tag{7}$$

$$\beta_{CH}^F = \frac{\sigma_s^2\left(\frac{\overline{f} - f_0}{\lambda} - \sigma_{sf}\right) - \sigma_{sf}\left(\frac{\overline{p}_s - p_c}{\lambda} - \sigma_s^2\right)}{\sigma_s^2\sigma_f^2 - \sigma_{sf}^2} \tag{8}$$

where the subscript "CH" denotes the "contracting-and-hedging strategy."

Therefore, given the information of risk aversion and prices $(\lambda, p_c, and\ f_0)$, (7) and (8) are useful formulas for calculating the optimal contracting and hedge ratios of the CH strategy.

Only Contracting As a comparison, an only-contracting strategy has $\beta^F = 0$. Solving (5) for the farmer's contracting ratio gives

$$\alpha_C^F = \frac{-\frac{\bar{p}_s - p_c}{\lambda} + \sigma_s^2}{\sigma_s^2} \tag{9}$$

where the subscript "C" denotes the "Only-contracting strategy". Without hedging, the contracting ratio is determined by the trade off between the risk premium and the spot price risk.

Only Hedging Another polar case is the only-hedging strategy. Having $\alpha^F = 0$ and solving (6) for the farmer's hedge ratio gives

$$\beta_H^F = \frac{1}{\sigma_f^2}\left(\frac{\bar{f} - f_0}{\lambda} - \sigma_{sf}\right) \tag{10}$$

where the subscript "H" denotes the "Only-hedging strategy". Without contracting, the hedge ratio is no longer affected by the risk premium, and the ratio becomes the familiar one in the hedging literature (Kahl, 1983). The traditional hedge ratio (10) is a special case of (8) when $\alpha^F = 0$. Comparing (8) and (10) reveals two effects of contracting upon the agent's hedging strategy. First, the revenue-risk structure is changed such that the denominator of (8) becomes more complicated. Second, the risk premium has entered the numerator and becomes an important factor influencing farmers' decision. Although a forward contract, like a futures contract, can lock in a forward price for the farmer, he has to pay a risk premium to the contractor. If the risk premium is low, contracting is more profitable and be promoted (hedging is depressed), and vice versa.

We might wonder whether α_{CH}^F is bigger than α_C^F, and whether β_{CH}^F is bigger than β_H^F. The answers depend on the magnitude of the futures price spread, the risk premium, and the vector of conditional price variances. Therefore, no signs can be predetermined without empirical study. A numerical example will demonstrate the relationship of these ratios.

3.2 The Meat Packer's Problem

Farmers are suppliers of hogs, and meat packers are buyers. Therefore, the packer's problem is the other side of a coin. Assume that there are many homogeneous packers who buy hogs as raw material. The packers have the hog price risk and the price risk of processed-goods. As in the farmer's problem, forward and futures contracts are two possible risk-management instruments. The representative packer has a profit function $\prod^p$ as follows:

$$\prod^p = p_z Z(q^p) - (1-\alpha^p)p_s q^p - \alpha^p p_c q^p + \beta^p q^p (f - f_0) \tag{11}$$

where α^p = the packer's contracting ratio, q^p = the packer's total number of hogs purchased, p_c = the contract price for hogs, $Z(\cdot)$ = the production function of meat processing with $Z'>0$ and $Z''<0$, and β^p = the packer's hedge ratio with negative numbers denoting short positions.

Using $q^p = 1$ (quantity normalization), (11) can be rewritten as

$$\prod^p = p_z Z - (1-\alpha^p)p_s - \alpha^p p_c + \beta^p (f - f_0) \tag{12}$$

Therefore, the mean and variance of the packer's profit are

$$E(\prod^p) = \bar{p}_z Z - (1-\alpha^p)\bar{p}_s - \alpha^p p_c + \beta^p(\bar{f} - f_0)$$
$$Var(\prod^p) = Z^2\sigma_z^2 + (1-\alpha^p)^2\sigma_s^2 + (\beta^p)^2\sigma_f^2$$

$$- 2Z(1-\alpha^p)\sigma_{Zs} + 2Z\beta^p\sigma_{Zf} - 2(1-\alpha^p)\beta^p\sigma_{sf}$$

where $\overline{p}_Z$ = the expected price for Z at delivery, σ_Z^2 = the variance of the price for Z, σ_{Zs} = the covariance of the processed-good and hog spot prices, and σ_{Zf} = the covariance of the processed-good and live-hog futures prices. The packer's problem is to choose the contracting ratio α^p and the hedge ratio β^p to maximize the expected utility Φ^p; that is,

$$\underset{\alpha^p \beta^p}{Max}\, \Phi^p = E(\prod^p) - \frac{\gamma}{2} Var(\prod^p)$$

The first-order conditions require

$$\overline{p}_s - p_c = \gamma\left[-(1-\alpha^p)\sigma_s^2 + Z\sigma_{Zs} + \beta^p\sigma_{sf}\right] \tag{13}$$

$$\overline{f} - f_0 = \gamma\left[\beta^p\sigma_f^2 - (1-\alpha^p)\sigma_{sf} + Z\sigma_{Zf}\right] \tag{14}$$

(13) shows that the packer should choose α^p such that the risk premium $\overline{p}_s - p_c$ equals the marginal change in risk by contracting, $-(1-\alpha^p)\sigma_s^2 + Z\sigma_{Zs} + \beta^p\sigma_{sf}$, adjusted by the coefficient of risk aversion γ. (14) shows that the packer should choose β^p such that the expected hedging profit (loss) $\overline{f} - f_0$ equals the marginal change in risk by hedging $\beta^p\sigma_f^2 - (1-\alpha^p)\sigma_{sf} + Z\sigma_{Zf}$, adjusted by γ. The first-order conditions are a bit more complicated than the farmer's. This is because the packer takes output price into consideration. Likewise, the second-order sufficient conditions for ensuring a maximum are shown to be satisfied in the appendix A. Furthermore, from the first-order conditions, the packer's optimal contracting and hedge strategies can be solved as

$$\alpha_{CH}^p = \frac{\sigma_f^2\left(\frac{\overline{p}_s - p_c}{\gamma} + \sigma_s^2 - \omega\sigma_{Zs}\right) - \sigma_{sf}\left(\frac{\overline{f} - f_0}{\gamma} + \sigma_{sf} - \omega\sigma_{Zf}\right)}{\sigma_s^2\sigma_f^2 - \sigma_{sf}^2} \tag{15}$$

$$\beta_{CH}^{p} = \frac{\sigma_s^2\left(\frac{\overline{f}-f_0}{\gamma} + \sigma_{sf} - \omega\sigma_{Zf}\right) - \sigma_{sf}\left(\frac{\overline{p}_s - p_c}{\gamma} + \sigma_s^2 - \omega\sigma_{Zs}\right)}{\sigma_s^2\sigma_f^2 - \sigma_{sf}^2} \tag{16}$$

where $\omega = \frac{Z}{q^p} = \frac{Z}{1} = Z$. Z is replaced by ω to avoid misleading. ω can be interpreted as the *dressed yield ratio per hog of packer's style* (Romans, 1974, pp. 102–103). Given the information of risk aversion and prices (γ, p_c, and f_0), (15) and (16) are useful formulas for calculating the optimal contracting and hedge ratios of the CH strategy.

Only Contracting The packer might use the only-contracting strategy to cope with his price risk. Using $\beta^p = 0$ and solving (13) for α_c^p gives the packer's contracting ratio as

$$\alpha_c^p = \frac{\frac{\overline{p}_s - p_c}{\gamma} + \sigma_s^2 - \omega\sigma_{Zs}}{\sigma_s^2} \tag{17}$$

Comparing (17) with the optimal contracting ratio of Haydu et al. (1992, equation (9)), we can find that they have exactly the same structure except for (17) without considering the interest rate.

Only Hedging The packer's another possible strategy is only-hedging. Using $\alpha^p = 0$ and solving (14) for β_H^p gives the packer's hedge ratio as

$$\beta_H^p = \frac{1}{\sigma_f^2}\left(\frac{\overline{f} - f_0}{\gamma} + \sigma_{sf} - \omega\sigma_{Zf}\right) \tag{18}$$

(18) is a bit more complicated than the traditional ratio in the literature. The difference is in the last term in the parentheses, $-\omega\sigma_{Zf}$. It implies that application of the traditional hedge ratio to the processing industry might commit an overestimation error due to ignoring the relationship between the

raw-material price and the processed-good price. The comparison between α^p_{CH} and α^p_C, and between β^p_{CH} and β^p_H, will also be demonstrated by the following numerical example.

4. A NUMERICAL EXAMPLE

A numerical example is used to demonstrate the application of the contracting and hedging model. There are three kinds of swine enterprises: farrow-to-feeder, feeder-to-finish, and farrow-to-finish. Different contracts have different contract lives. A feeder-to-finish contract is four months long, and a farrow-to-finish contract is ten. The time length is associated the time needed to finish hogs from feeder or farrow stages. The example is a feeder-to-finish contract. Therefore, the prices at decision time are four-month earlier than those at delivery time. Besides, the manipulation of the futures price is also affected by the time lag. For example, to hedge hog price risk in February, an agent may use the June contract.

4.1 Data Sources

A monthly data set was collected for the numerical example. The time horizon is from January 1985 to December 1994, a total of 120 observations. Spot price variables were obtained from the U.S. Department of Agriculture (USDA) annual publication, *Agricultural Prices*, and *Livestock and Meat Statistics* supplement *Red Meats Yearbook* (1995). The hog spot price is the barrow and gilt 5/6/7 market average price. The pork price is the wholesale value for pork. Futures price variables were obtained from Tick Data Inc. (Lakewood, Colorado) for early observations and *Feedstuffs* for more recent observations.

4.2 Estimation of Parameters

To calculate the various hedge and contracting ratios, we need the information of expected forward prices at delivery, and the conditional covariance matrix. Therefore, the following regressive models are used to estimate these parameters.

$$LHSP = \theta_0+\theta_1 LHSP1+\theta_2 LHFP1+\theta_3 PKP1+\theta_4 LHSP2+\theta_5 LHFP2 +\theta_6 PKP2+\theta_7 LHSPY+\theta_8 LHFPY+\theta_9 PKPY+\varepsilon_s \quad (19)$$

$$LHFP = \theta_0+\theta_1 LHSP1+\theta_2 LHFP1+\theta_3 PKP1+\theta_4 LHSP2+\theta_5 LHFP2 +\theta_6 PKP2+\theta_7 LHSPY+\theta_8 LHFPY+\theta_9 PKPY+\varepsilon_f \quad (20)$$

$$PKP = \theta_0+\theta_1 LHSP1+\theta_2 LHFP1+\theta_3 PKP1+\theta_4 LHSP2+\theta_5 LHFP2 +\theta_6 PKP2+\theta_7 LHSPY+\theta_8 LHFPY+\theta_9 PKPY+\varepsilon_z \quad (21)$$

where LHSP, LHFP, and PKP denote the live-hog spot price, live-hog futures price, and pork price, respectively; '1', '2', and 'Y' denote: 'one period lag', 'two period lag', and 'one year lag', respectively; the length of one period is 4 months for the feeder-to-finish contract. Obviously, the structure of the model is a three-equation vector autoregressive system, or VAR(3) (Haydu et al., 1992) where LHSP, LHFP, and PKP are assumed to be functions of lagged variables of each price. It means that one uses available information at the decision time to forecast prices. Including one-year-lag variables, i.e. LHSPY, LHFPY, and PKPY, is for seasonality. Since the error terms of VAR(3) may not have properties of vector white noise, such that the OLS procedure is not appropriate. Myers and Thompson (1989) suggested to estimate the system by SUR. After the equations are estimated, the estimated residuals can be used to calculate the estimated conditional covariance matrix for prices. The results are presented in appendix B and they are corrected for the first-order autocorrelation by the Cochrane-Orcutt iterative procedure (Gujarati, 1988, pp. 383–4). The χ^2 value for the contemporaneous correlation test (Judge, 1988, p.456) is 258.37. The critical value is $\chi^2(3)=11.3449$. Therefore, the null hypothese is rejected. It means that the use of SUR is justified. Using the results, the expected hog spot price, hog futures price, and pork spot price can be calculated as

$$(\hat{p}_s, \hat{f}, \hat{p}_z) = (47.595, 47.541, 46.530)$$

and the expected price variance matrix as

$$\begin{bmatrix} \hat{\sigma}_s^2 & \hat{\sigma}_{sf} & \hat{\sigma}_{zs} \\ \hat{\sigma}_{sf} & \hat{\sigma}_f^2 & \hat{\sigma}_{zf} \\ \hat{\sigma}_{zs} & \hat{\sigma}_{zf} & \hat{\sigma}_z^2 \end{bmatrix} = \begin{bmatrix} 8.221 & 5.815 & 10.186 \\ 5.815 & 12.012 & 7.244 \\ 10.186 & 7.244 & 17.848 \end{bmatrix}$$

In addition, we need to know the values of the dressed yield ratio (ω) and risk premium ratio ($r = (p_s - p_c)/p_s$). According to USDA/NASS *Livestock Slaughter*, the 1994 average live weight and dressed weight of hog slaughtered under Federal inspection are 256 lb. and 180 lb., respectively. Therefore, ω is estimated as 70% (=180/256). r is assumed to range from 2% to 8% which is equivalent to from \$0.952/cwt to \$3.808/cwt. The equivalent contract price will range from \$46.645/cwt to \$43.787/cwt. These numbers are close to those used in Fleming (1995).

4.3 The Farmer's Strategies and Evaluation

The estimates of the farmer's contracting ratio, hedge ratio, and expected utility with respect to the coefficient of risk aversion (λ) and the risk premium ratio (r) are presented in table 1. Columns 3–6 are the ratios in different strategies. Columns 7–10 are the expected utility. The third and fourth numbers in the first row show that the optimal contracting ratio ($\hat{\alpha}_{CH}^F$) and

Table 1 The Farmer's Contracting Ratios, Hedge Ratios, and Expected Utility

		CH Strategy		C Strategy	H Strategy	Expected Utility			
r	λ	$\hat{\alpha}^F_{CH}$	$\hat{\beta}^F_{CH}$	$\hat{\alpha}^F_C$	$\hat{\beta}^F_H$	EUCH	EUC	EUH	EUNO
0.02	2	0.957	0.021	0.942	-0.442	46.821	46.593	39.210	39.374
	4	0.979	0.011	0.971	-0.463	46.732	46.618	30.947	31.153
	6	0.986	0.007	0.981	-0.470	46.702	46.626	22.711	22.932
	8	0.989	0.005	0.986	-0.474	46.687	46.631	14.483	14.711
0.04	2	0.869	-0.021	0.884	-0.442	45.955	45.725	39.661	39.374
	4	0.935	-0.011	0.942	-0.463	45.823	45.708	31.420	31.153
	6	0.956	-0.007	0.961	-0.470	45.779	45.702	23.192	22.932
	8	0.967	-0.005	0.971	-0.474	45.757	45.700	14.968	14.711
0.06	2	0.781	-0.064	0.826	-0.442	45.174	44.897	40.067	39.374
	4	0.891	-0.032	0.913	-0.463	44.957	44.818	31.871	31.153
	6	0.927	-0.021	0.942	-0.470	44.884	44.792	23.658	22.932
	8	0.945	-0.016	0.957	-0.474	44.848	44.779	15.441	14.711
0.08	2	0.693	-0.106	0.768	-0.442	44.479	44.11	40.429	39.374
	4	0.847	-0.053	0.884	-0.463	44.133	43.949	32.300	31.153
	6	0.898	-0.035	0.923	-0.470	44.018	43.895	24.109	22.932
	8	0.923	-0.027	0.942	-0.474	43.960	43.868	15.904	14.711

optimal hedge ratio ($\hat{\beta}^F_{CH}$) for a farmer with $\lambda = 2$ are 0.957 and 0.021 (a positive number stands for a long position), when he uses the contracting-and-hedging strategy (CH). However, the contracting ratio of the only-contracting strategy (C), $\hat{\alpha}^F_C$, is lower or 0.942. The hedge ratio of the only-hedging strategy (H), $\hat{\beta}^F_H$, changes its sign as -0.442 (a short position). Furthermore, the order of expected utility is:

$$EUCH > EUC > EUH > EUNO$$

where "EU" denotes for expected utility, and "NO" denotes no instrument used for risk-management. Therefore, if given the choice, and no transaction costs, the farmer will select the CH strategy. However, if the transaction cost of hedging, say commission and interest cost, is greater than \$0.228/cwt (= 46.821 – 46.593), then the C strategy is better than CH. Given r, the rise in λ will cause $\hat{\alpha}^F_{CH}$ to increase, $\hat{\beta}^F_{CH}$ to decrease, $\hat{\alpha}^F_C$ to increase, and $\hat{\beta}^F_H$ to increase. It means that the agent tends to take more positions in risk-management tools, when he becomes more risk averse. However, since contracting and hedging are substitutes for the farmer, $\hat{\alpha}^F_{CH}$ and $\hat{\beta}^F_{CH}$ of the CH strategy move in opposite directions. Given λ, $\hat{\alpha}^F_{CH}$ decreases, $\hat{\beta}^F_{CH}$ increases (toward more short positions), $\hat{\alpha}^F_C$ decreases, and $\hat{\beta}^F_H$ stays unchanged as r increases. It means that when the risk premium ratio raises, the incentive for contracting is depressed, while the incentive for hedging of the CH strategy is raised. The farmer with the H strategy does not contract; therefore, $\hat{\beta}^F_H$ is not effected by the change in r. In addition, when r is higher than 0.04, all hedge ratios become negative. This means that the farmer with the CH strategy will take short positions in live-hog futures in most of cases. However, an extremely low r will cause the farmer to take a long position. Nevertheless, the ratios are close to naught. Although most of ratios change along with r and λ, the relations among the ratios and the order of the expected utilities stay the same.

4.4 The Meat Packer's Strategies and Evaluation

Table 2 presents the simulation results for the meat packer. The third and fourth numbers in the first row show that the optimal contracting ratio ($\hat{\alpha}^P_{CH}$) and optimal hedge ratio ($\hat{\beta}^P_{CH}$) for a meat packer with $\gamma = 0.2$ are

Table 2 The Meat Packer's Contracting Ratios, Hedge Ratios, and Expected Utility

		CH Strategy		C Strategy	H Strategy	Expected Utility			
r	γ	$\hat{\alpha}^P_{CH}$	$\hat{\beta}^P_{CH}$	$\hat{\alpha}^P_C$	$\hat{\beta}^P_H$	EUCH	EUC	EUH	EUNO
0.02	0.2	0.563	0.210	0.712	0.483	38.554	22.978	33.150	27.890
	0.4	0.349	0.103	0.422	0.272	33.036	25.249	30.264	27.620
	0.6	0.278	0.068	0.326	0.202	31.026	25.835	29.121	27.349
	0.8	0.242	0.050	0.277	0.167	29.893	26.000	28.414	27.079
0.04	0.2	1.000	-0.216	1.000	0.483	39.349	24.036	32.736	27.890
	0.4	0.789	-0.110	0.712	0.272	33.498	25.840	30.023	27.620
	0.6	0.571	-0.074	0.519	0.202	31.376	26.271	28.938	27.349
	0.8	0.462	-0.057	0.422	0.167	30.188	26.358	28.260	27.079
0.06	0.2	1.000	-0.642	1.000	0.483	40.998	25.503	31.878	27.890
	0.4	1.000	-0.323	1.000	0.272	34.386	26.636	29.561	27.620
	0.6	0.865	-0.216	0.712	0.202	32.012	26.843	28.608	27.349
	0.8	0.682	-0.163	0.567	0.167	30.696	26.819	27.996	27.079
0.08	0.2	1.000	-1.068	1.000	0.483	43.499	27.379	30.575	27.890
	0.4	1.000	-0.536	1.000	0.272	35.701	27.637	28.876	27.620
	0.6	1.000	-0.358	0.905	0.202	32.931	27.552	28.129	27.349
	0.8	0.902	-0.270	0.712	0.167	31.418	27.381	27.620	27.079

0.563 and 0.210 (a long position), when he uses the CH strategy. The contracting ratio of the C strategy ($\hat{\alpha}^P_C$) is higher or 0.712. The hedge ratio of the H strategy ($\hat{\beta}^P_H$) is also higher or 0.483. Furthermore, the order of expected utility is:

$$EUCH > EUH > EUNO > EUC$$

Therefore, the CH strategy is the optimal strategy for it makes the highest expected utility. It is noticeable that the NO strategy is better than the C strategy. It means that if the packer contracts with a farmer and doesn't hedge his extra price risk from contracting by some instruments, then the C strategy will make him worse off than before.

When the risk premium ratio (r) is greater than 4%, the packer with the CH strategy will change his hedging position from long to short. This is because the packer will contract more hogs when r increases. After having the contract hogs up to a substantial number, the packer's will behave like a farmer owning hogs and will take short position in futures. Given r, as γ increases, all $\hat{\alpha}^P_{CH}$, $\hat{\beta}^P_{CH}$, $\hat{\alpha}^P_C$, and $\hat{\beta}^P_H$ decreases in absolute values. It means that when the packer becomes more risk averse, the ratios used to manage risk will be lower. The result that both $\hat{\alpha}^P_{CH}$ and $\hat{\beta}^P_{CH}$ decrease as γ increases supports the proposition of hedging and contracting being complements for the packer. Although most of the ratios changes along with r and γ, the relationships among ratios and the order of expected utility stay the same.

5. CONCLUSION

This paper presented a theoretical model of contracting and hedging. Under the framework, the farmer's and the meat packer's contracting ratios and hedge ratios were derived. A numerical example for the hog industry was used for demonstrating the application of the ratios. The results indicated that contracting and hedging are competing marketing instruments for farmers, but complements for packers. The slim increase in expected utility of farmers with contracting-and-hedging strategy over those with only-contracting shows that farmers should probably market their hogs completely on contract when transaction costs are considered. On the other hand, in most cases, the expected utilities of packers with contracting-and-hedging strategy, and none strategy were ordered as the first and second positions. It means that packers were supposed to take either both or neither marketing instruments for their best advantage.

Appendix A

Second-Order Conditions for the Farmer's Problem

Taking a differential of the first-order conditions (associated with (5) and (6)) with respect to α^F and β^F gives a Hessian matrix as

$$\Delta^F = \begin{bmatrix} -\lambda\sigma_s^2 & \lambda\sigma_{sf} \\ \lambda\sigma_{sf} & -\lambda\sigma_f^2 \end{bmatrix} \tag{A–1}$$

Therefore, the second-order sufficient conditions are

$$\left|\Delta^F\right| > 0, \ -\lambda\sigma_f^2 < 0, \text{ and } -\lambda\sigma_s^2 < 0$$

These conditions can be simplified as

$$\sigma_s^2\sigma_f^2 - \sigma_{sf}^2 > 0 \text{ or } \sigma_s^2\sigma_f^2(1 - \rho_{sf}^2) > 0$$

where ρ_{sf} = the correlation coefficient of spot and futures prices. Clearly, the inequality is always true, because the absolute value of a correlation coefficient is supposed to be less than one.

Second-Order Conditions for the Packer's Problem

Taking a differential of the first-order conditions (associated with (13) and (14)) with respect to α^P and β^P gives a Hessian matrix as

$$\Delta^P = \begin{bmatrix} -\gamma\sigma_s^2 & -\gamma\sigma_{sf} \\ -\gamma\sigma_{sf} & -\gamma\sigma_f^2 \end{bmatrix} \tag{A–2}$$

The second-order sufficient conditions are

$$|\Delta^P| > 0, \ -\gamma\sigma_f^2 < 0, \ \text{and} \ -\gamma\sigma_s^2 < 0$$

As for the farmer's problem, these conditions are always satisfied.

Appendix B

SUR Results for the LHSP, LHFP and PKP Equations

Independent Variables	Dependent Variables		
	LHSP	LHFP	PKP
CONST	0.546**[a]	0.097*	1.149**
	(0.129)	(0.043)	(0.244)
LHSP1	0.163	0.090	0.298
	(0.123)	(0.159)	(0.189)
LHFP1	-0.057	0.175*	-0.146
	(0.064)	(0.082)	(0.097)
PKP1	-0.165*	0.036	-0.175
	(0.082)	(0.101)	(0.126)
LHSP2	-0.037	-0.045	-0.202
	(0.123)	(0.159)	(0.188)
LHFP2	-0.007	-0.002	0.037
	(0.062)	(0.079)	(0.095)
PKP2	0.035	0.106	0.052
	(0.083)	(0.102)	(0.127)
LHSPY	0.502**	-0.376*	0.380*
	(0.116)	(0.148)	(0.179)
LHFPY	-0.057	0.387**	-0.067
	(0.050)	(0.066)	(0.076)
PKPY	-0.149	0.104	-0.068
	(0.079)	(0.093)	(0.122)

a Double asterisk (**) indicates statistically significant at the 1% level; single asterisk (*) indicates statistically significant at the 5% level.

Footnotes

1. Interest and transaction costs are neglected in (1), although they are suggested to be significant by Shapiro & Brorsen (1988) and Williams (1987). We do not mean to deny that interest and transaction costs might be important factors influencing farmers' decisions. However, if we take interest and transaction costs as well as price rigidity into the model, it might become too complicated such that no clear insight can be gained. Therefore, we do not consider these two items of costs here.

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契約運銷與期貨避險：理論與應用—以美國毛豬產業為例

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本文主要在探討農戶和肉類加工廠如何搭配運銷契約與活豬期貨契約，來管理其價格風險。在資產組合理論「均數 —— 變異數分析」的架構下，本模型推導出最適契約比率和最適期貨避險比率。本文並使用一組美國「小豬至成豬型」飼養業的資料來模擬如何應用這些最適比率在農戶和肉類加工廠的決策上。在「預期效用」的基準上，比較「同時使用兩種工具」、「只有使用期貨避險」、和「只有使用運銷契約」等三種策略的績效。結果顯示：在不考慮交易成本的情況下，「同時使用兩種工具」的策略為最佳。並且我們發現運銷契約和期貨契約對農戶來說是替代的工具；對肉類加工廠來說是互補的工具。

關鍵詞：遠期契約、期貨避險、風險、小豬至成豬型飼養業